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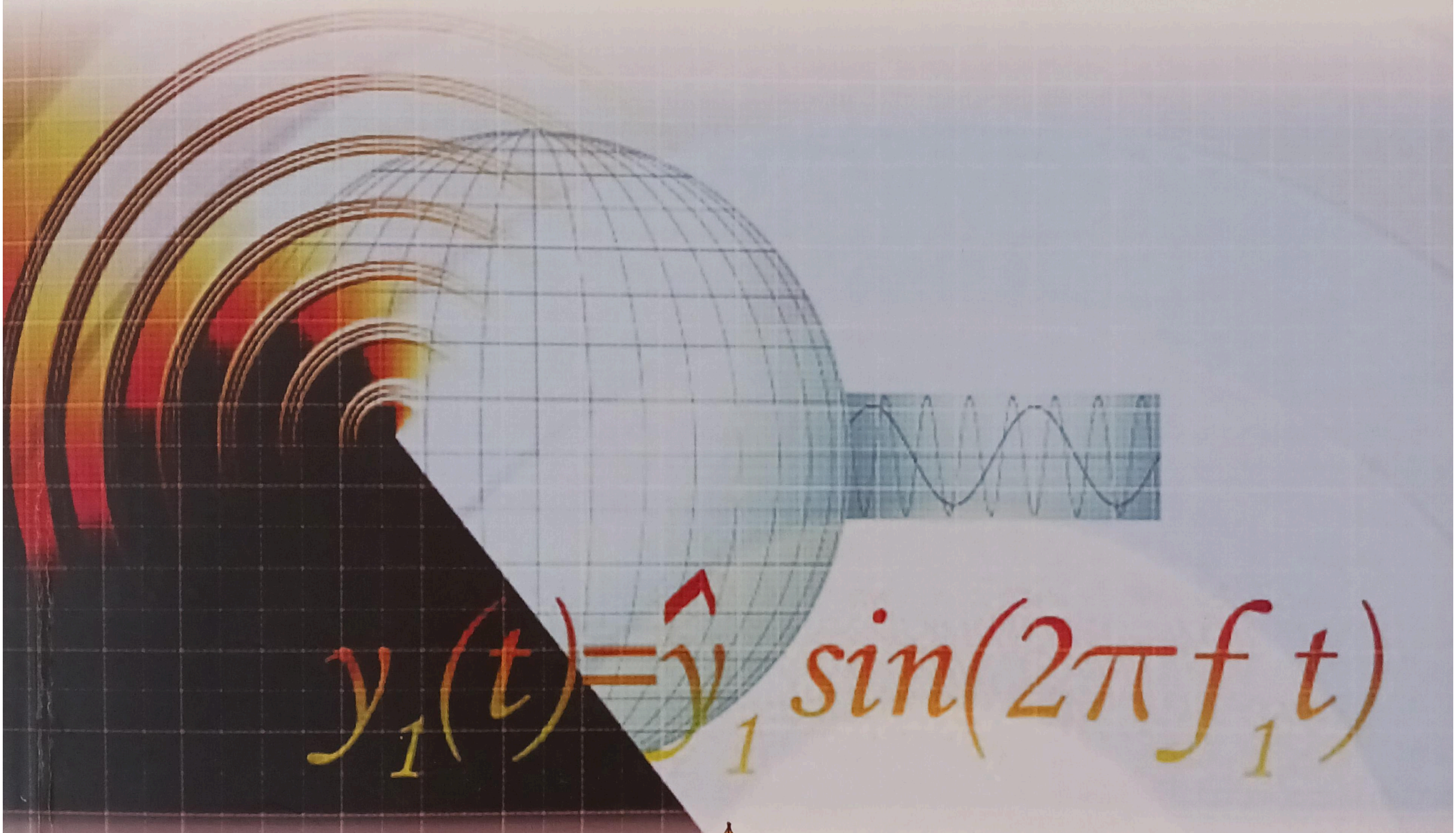


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# EFFECT OF MICHELL FUNCTION ON THE THICKNESS OF ANNULAR DISC WITH INTERNAL HEAT GENERATION

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**Abstract**

The present paper deals with the determination of displacement and thermal stresses in a thick $(M \neq 0)$ annular disc with internal heat generation. Arbitrary heat $f(r)$ is applied on the upper surface of disc, whereas lower surface dissipates heat by convection and the fixed circular edges are thermally insulated. Here we compute the effects of Michell function of a thick annular disc in terms of stresses along radial direction. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.

**Keywords:** Thick annular disc $(M \neq 0)$, Thin annular disc $(M = 0)$, internal heat generation.

## Introduction

Deshmukh [1] studied transient heat conduction problem in a thin hollow cylinder and determined thermal stresses. Gogulwar and Deshmukh [2] studied the inverse problem of thermal stresses in a thin annular disc. Kulkarni and Deshmukh [3] has determined the quasi-static steady state thermal stresses in thick annular disc. Kulkarni et al. [4] has determined displacement and thermal stresses in thin hollow circular disc due to internal heat generation within it. Shang Sheng Wu [8] studied the direct thermoelastic problem in an annular fin with its base subjected to a heat flux of a decayed exponential function of time. In this paper thick $(M \neq 0)$ and thin $(M = 0)$ annular disc is considered and discussed its thermoelasticity with the help of the Goodier's thermoelastic displacement potential function and the Michell's function. To obtain the temperature distribution integral transform method is applied. The results are obtained in series form in terms of Bessel's functions and the temperature change, displacement function and stresses have been computed numerically and illustrated graphically. Here we compute the effects of Michell function in terms of stresses along radial direction. A mathematical model has been constructed of a thick $(M \neq 0)$ and thin $(M = 0)$ annular disc with the help of numerical illustration by considering copper (pure) plate. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant, gas power plant and the measurement of aerodynamic heating.

**Formulation of the Problem**

Consider a thick ($M \neq 0$) annular disc of thickness 2h defined by $a \leq r \leq b, -h \leq z \leq h$. An arbitrary heat $f(r)$ is applied on the upper surface of the disc ( $z = h$ ) and heat dissipates by convection from the lower boundary surface (z = - h) into the surrounding at the zero temperature. The circular edge ($r = a$ and $r = b$ ) are thermally insulated. Assume that the boundary of the annular disc is free from traction. Under these prescribed conditions, the quasi-static steady state thermal stresses are required to be determined.

The differential equation governing the displacement potential function $\phi(r, z)$ is given in [5] as

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = K\tau \tag{1}$$

where K is the restraint coefficient and temperature change $\tau = T - T_{i,}$ $T_i$ is initial(ambient) temperature. Displacement function $\phi$ is known as Goodier's thermoelastic displacement potential.

The steady state temperature of the plate satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0 \tag{2}$$

with the boundary conditions

$$\frac{\partial T}{\partial r} = 0 \qquad at\ r = a, \quad -h \leq z \leq h \tag{3}$$

$$\frac{\partial T}{\partial r} = 0 \qquad at\ r = b, \quad -h \leq z \leq h \tag{4}$$

$$\frac{\partial T}{\partial z} + h_{s_1}T = f(r) \qquad at\ z = h, a \leq r \leq b \tag{5}$$

$$\frac{\partial T}{\partial z} - h_{s_2}T = 0 \qquad at\ z = -h, a \leq r \leq b \tag{6}$$

where k is the thermal conductivity of the material of the plate, q is internal heat generation, $h_{s_1}$ and $h_{s_2}$ are the relative heat transfer coefficients on the upper and lower surface of the thick annular disk.

The Michell's function M must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{7}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The components of the stresses are represented by the thermoelastic displacement potential $\phi$ and Michell's function M as

$$\sigma_{rr} = 2G\left\{\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left[\nu\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{8}$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left[\nu\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right]\right\} \tag{9}$$

$$\sigma_{zz} = 2G\left\{\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{10}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{11}$$

where, G and v are the shear modulus and Poisson's ratio respectively.

The boundary conditions on the traction free surface of an annular disc are

$$\sigma_{rr} = \sigma_{rz} = 0$$

at $r = a,\ z = \pm h$ (12)

**Solution of the Heat Conduction Equation**

To obtain the expression for temperature $T(r, z)$, we introduce the finite Hankel transform over the variable r and its inverse transform defined as in [6]

$$\bar{T}(\beta_m, z) = \int_{r=a}^{b} r\, K_0(\beta_m, r)\; T(r, z) \tag{13}$$

$$T(r, z) = \sum_{m=1}^{\infty} K_0(\beta_m, r)\; \bar{T}(\beta_m, z) \tag{14}$$

where, $K_0(\beta_m, r) = \frac{R_0(\beta_m, r)}{\sqrt{N}}$, (15)

$$R_0(\beta_m, r) = \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)} \tag{16}$$

The normality constant

$$N = \frac{b^2}{2} R_0(\beta_m, b)^2 - \frac{a^2}{2} R_0(\beta_m, a)^2 \tag{17}$$

and $\beta_1, \beta_2 \ldots..$ are roots of the transcendental equation

$$\frac{J_1(\beta_m a)}{J_1(\beta_m b)} - \frac{Y_1(\beta_m a)}{Y_1(\beta_m b)} = 0 \tag{18}$$

where, $J_n(x)$ is Bessel function of the first kind of order n and $Y_n(x)$ is Bessel function of the second kind of order n.

On applying the finite Hankel transform defined in the Eq. (13) and its inverse transform defined in Eq. (14) to the Eq. (2), one obtains the expression for temperature as

$$T(r,z) = \sum_{m=1}^{\infty} \frac{1}{\sqrt{N}} \left[ \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)} \right] \left\{ - \frac{1}{(\beta_m{}^2 + h_{s_1} h_{s_2}) \sinh(2\beta_m \mathrm{h}) + \beta_m (h_{s_1} + h_{s_2}) \cosh(2\beta_m \mathrm{h})} \right.$$

$$\times \left[ \begin{array}{l} \left( \frac{\mathrm{dA}(\beta_m, \mathrm{h})}{\mathrm{dZ}} + h_{s_1} \mathrm{A}(\beta_m, \mathrm{h}) - \mathrm{F}(\beta_m) \right) \langle \beta_m \cosh[\beta_m (z + \mathrm{h})] + h_{s_2} \sinh[\beta_m (z + \mathrm{h})] \rangle \\ + \left( \frac{\mathrm{dA}(\beta_m, -\mathrm{h})}{\mathrm{dZ}} - h_{s_2} \mathrm{A}(\beta_m, -\mathrm{h}) \right) \langle -\beta_m \cosh[\beta_m (z - \mathrm{h})] + h_{s_1} \sinh[\beta_m (z - \mathrm{h})] \rangle \end{array} \right]$$

$$\left. + \mathrm{A}(\beta_m, z) \right\} \tag{19}$$

$A(\beta_m, z)$ is particular integral of differential equation (2) and $F(\beta_m)$ is the Hankel transform of f(r).

$$F(\beta_m) = \int_{r=a}^{b} \frac{r}{\sqrt{N}} \left[ \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)} \right] f(r)\, dr \tag{20}$$

**Michells function M**

Now suitable form of M which satisfy Eq. (6) is given by

$$M = K \sum_{m=1}^{\infty} \frac{F(\beta_m)}{\sqrt{N}} \left[ \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)} \right]$$

$$\times \left\{ B_m \left( \begin{array}{c} \beta_m \cosh[\beta_m (z + \mathrm{h})] \\ + h_{s_2} \sinh[\beta_m (z + \mathrm{h})] - \beta_m e^{\beta_m (z + \mathrm{h})} \end{array} \right) \right.$$

$$\left. + C_m \beta_m (z + \mathrm{h}) \left( \begin{array}{c} -\beta_m \cosh[\beta_m (z - \mathrm{h})] + h_{s_1} \sinh[\beta_m (z - \mathrm{h})] \\ + \cosh(2\beta_m \mathrm{h})\, \beta_m e^{\beta_m (z + \mathrm{h})} + h_{s_1} \sinh(2\beta_m \mathrm{h})\, e^{\beta_m (z + \mathrm{h})} \end{array} \right) \right\} \tag{21}$$

where, $B_m$ and $C_m$ are arbitrary functions.

**Goodiers Thermoelastic Displacement Potential $\phi$**

Assuming the displacement function $\phi(r,z)$ which satisfies Eq. (1) as

$$\phi(r,z) = \sum_{m=1}^{\infty} \left\{ \frac{K}{\sqrt{N} [(\beta_m{}^2 + h_{s_1} h_{s_2}) \sinh(2\beta_m \mathrm{h}) + \beta_m (h_{s_1} + h_{s_2}) \cosh(2\beta_m \mathrm{h})]} \left[ \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)} \right] \right.$$

$$\times \quad - \left( \frac{\mathrm{dA}(\beta_m, \mathrm{h})}{\mathrm{dZ}} + h_{s_1} \mathrm{A}(\beta_m, \mathrm{h}) - \mathrm{F}(\beta_m) \right)$$

$$\times \begin{bmatrix} \beta_m \cosh[\beta_m (z+h)] + h_{s_2} \sinh[\beta_m (z+h)] \\ -\beta_m e^{\beta_m (z+h)} \end{bmatrix} - \left( \frac{dA(\beta_m, -h)}{dZ} - h_{s_2} A(\beta_m, -h) \right)$$

$$\times \begin{bmatrix} -\beta_m \cosh[\beta_m (z-h)] + h_{s_1} \sinh[\beta_m (z-h)] \\ +\beta_m \cosh(2\beta_m h)\, e^{\beta_m (z+h)} + h_{s_1} \sinh(2\beta_m h)\, e^{\beta_m (z+h)} \end{bmatrix} \Bigg\} \qquad (22)$$

Now using Eqs. (19), (21) and (22) in Eq. (8), (9), (10) & (11), one obtains the expressions for stresses respectively as

We set for convenience

$$g_1(r) = \frac{J_1'(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_1'(\beta_m r)}{\beta_m Y_0'(\beta_m b)},$$

$$g_2(r) = \frac{J_0(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0'(\beta_m b)},$$

$$g_3(r) = \frac{J_0'(\beta_m r)}{\beta_m J_0'(\beta_m b)} - \frac{Y_0'(\beta_m r)}{\beta_m Y_0'(\beta_m b)},$$

$$U = [(\beta_m{}^2 + h_{s_1} h_{s_2}) \sinh(2\beta_m h) + \beta_m (h_{s_1} + h_{s_2}) \cosh(2\beta_m h)],$$

$$N = \frac{dA(\beta_m, h)}{dZ} + h_{s_1} A(\beta_m, h) - F(\beta_m),$$

$$L = \frac{dA(\beta_m, -h)}{dZ} - h_{s_2} A(\beta_m, -h),$$

$$u_1 = [J_0(\beta_m a) Y_0'(\beta_m b) - Y_0'(\beta_m a) J_0'(\beta_m b)],$$

$$\text{and } v_1 = [J_1'(\beta_m a) Y_0'(\beta_m b) - Y_1'(\beta_m a) J_0'(\beta_m b)]$$

$$\frac{\sigma_{rr}}{K} = 2G \sum_{m=1}^{\infty} \Bigg\{ \frac{g_1(r)}{U\sqrt{N}} N[\beta_m \cosh[\beta_m (z+h)] + h_{s_2} \sinh[\beta_m (z+h)] - \beta_m e^{\beta_m (z+h)}]$$

$$+ L \begin{bmatrix} -\beta_m \cosh[\beta_m (z-h)] + h_{s_1} \sinh[\beta_m (z-h)] \\ +\beta_m \cosh(2\beta_m h)\, e^{\beta_m (z+h)} + h_{s_1} \sinh(2\beta_m h)\, e^{\beta_m (z+h)} \end{bmatrix}$$

$$+ U F(\beta_m) \begin{bmatrix} B_m \left( \beta_m \sinh[\beta_m (z+h)] + h_{s_2} \cosh[\beta_m (z+h)] - \beta_m e^{\beta_m (z+h)} \right) \\ + C_m \beta_m \begin{pmatrix} -\beta_m \cosh[\beta_m (z-h)] + h_{s_1} \sinh[\beta_m (z-h)] \\ +\beta_m \cosh(2\beta_m h)\, e^{\beta_m (z+h)} + h_{s_1} \sinh(2\beta_m h)\, e^{\beta_m (z+h)} \end{pmatrix} \\ + C_m \beta_m{}^2 (z+h) \begin{pmatrix} -\beta_m \sinh[\beta_m (z-h)] + h_{s_1} \cosh[\beta_m (z-h)] \\ +\beta_m \cosh(2\beta_m h)\, e^{\beta_m (z+h)} + h_{s_1} \sinh(2\beta_m h)\, e^{\beta_m (z+h)} \end{pmatrix} \end{bmatrix}$$

$$+ \frac{g_2(r)}{U\sqrt{N}} \; N[\beta_m \cosh[\beta_m (z+h)] + h_{s_2} \sinh[\beta_m (z+h)] - \beta_m e^{\beta_m (z+h)}]$$

$$+L\left[-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\right] + A(\beta_m, z)$$

$$+2v\,U\,C_m\,\beta_m{}^3 F(\beta_m)\left[\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right]\Bigg]\Bigg\} \quad (23)$$

$$\frac{\sigma_{\theta\theta}}{K} = 2G\sum_{m=1}^{\infty}\left\{\frac{g_3(r)}{rU\sqrt{N}}\left[-N\left[\beta_m\cosh[\beta_m(z+h)] + h_{s_2}\sinh[\beta_m(z+h)] - \beta_m e^{\beta_m(z+h)}\right]\right.\right.$$

$$-L\left[\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right]$$

$$-U\,F(\beta_m)\left[\begin{matrix}B_m\left(\beta_m\sinh[\beta_m(z+h)] + h_{s_2}\cosh[\beta_m(z+h)] - \beta_m e^{\beta_m(z+h)}\right)\\ +C_m\,\beta_m\left(\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right)\\ +C_m\,\beta_m{}^2(z+h)\left(\begin{matrix}-\beta_m \sinh[\beta_m(z-h)] + h_{s_1}\cosh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right)\end{matrix}\right]\Bigg]$$

$$+\frac{g_2(r)}{U\sqrt{N}}\left[N\left[\beta_m\cosh[\beta_m(z+h)] + h_{s_2}\sinh[\beta_m(z+h)]\right]\right.$$

$$+L\left[-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\right] + A(\beta_m, z)$$

$$+2v\,U\,C_m\,\beta_m{}^3 F(\beta_m)\left[\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right]\Bigg]\Bigg\} \quad (24)$$

$$\frac{\sigma_{zz}}{K} = 2G\sum_{m=1}^{\infty}\frac{g_2(r)}{U\sqrt{N}}\left\{-N\beta_m{}^2\left[\beta_m\cosh[\beta_m(z+h)] + h_{s_2}\sinh[\beta_m(z+h)] - \beta_m e^{\beta_m(z+h)}\right]\right.$$

$$-L\beta_m{}^2\left[\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right]$$

$$+N\left[\beta_m\cosh[\beta_m(z+h)] + h_{s_2}\sinh[\beta_m(z+h)]\right]$$

$$+L\left[-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\right] + A(\beta_m, z)$$

$$+UF(\beta_m)\left[\begin{matrix}(1-v)2\,C_m\,\beta_m{}^3\left(\begin{matrix}-\beta_m \cosh[\beta_m(z-h)] + h_{s_1}\sinh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right)\\ -B_m\,\beta_m{}^3\left(\beta_m\sinh[\beta_m(z+h)] + h_{s_2}\cosh[\beta_m(z+h)] - \beta_m e^{\beta_m(z+h)}\right)\\ +C_m\,\beta_m{}^4(z+h)\left(\begin{matrix}-\beta_m \sinh[\beta_m(z-h)] + h_{s_1}\cosh[\beta_m(z-h)]\\ +\beta_m\cosh(2\beta_m h)\,e^{\beta_m(z+h)} + h_{s_1}\sinh(2\beta_m h)\,e^{\beta_m(z+h)}\end{matrix}\right)\end{matrix}\right]\Bigg\} \quad (25)$$

$$\frac{\sigma_{rz}}{K} = 2G\sum_{m=1}^{\infty}\frac{g_3(r)}{U\sqrt{N}}\left\{-N\beta_m\left[\beta_m\sinh[\beta_m(z+h)] + h_{s_2}\cosh[\beta_m(z+h)] - \beta_m e^{\beta_m(z+h)}\right]\right.$$

$$-\beta_m L \begin{bmatrix} -\beta_m \sinh[\beta_m (z-\mathrm{h})] + h_{s_1} \cosh[\beta_m (z-\mathrm{h})] \\ +\beta_m \cosh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} + h_{s_1} \sinh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} \end{bmatrix}$$

$$+ U\beta_m{}^2 F(\beta_m) \left[ \begin{array}{l} -2\nu\, C_m \begin{pmatrix} -\beta_m \sinh[\beta_m (z-\mathrm{h})] + h_{s_1} \cosh[\beta_m (z-\mathrm{h})] \\ +\beta_m \cosh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} + h_{s_1} \sinh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} \end{pmatrix} \\ - C_m (z+\mathrm{h}) \begin{pmatrix} -\beta_m \cosh[\beta_m (z-\mathrm{h})] + h_{s_1} \sinh[\beta_m (z-\mathrm{h})] \\ +\beta_m \cosh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} + h_{s_1} \sinh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} \end{pmatrix} \\ - B_m \begin{pmatrix} -\beta_m \cosh[\beta_m (z-\mathrm{h})] + h_{s_1} \sinh[\beta_m (z-\mathrm{h})] \\ +\beta_m \cosh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} + h_{s_1} \sinh(2\beta_m \mathrm{h})\, e^{\beta_m (z+\mathrm{h})} \end{pmatrix} \end{array} \right] \Bigg\} \tag{26}$$

In order to satisfy condition (12), solving equations (23) and (26) for $B_m$ and $C_m$, one obtains

$$B_m = \frac{u_1}{UF(\beta_m)(\beta_m - h_{s_2})v_1}$$

$$\times \left[N\beta_m + L(-\beta_m \cosh(2\beta_m \mathrm{h}) - h_{s_1} \sinh(2\beta_m \mathrm{h}) + A(\beta_m, -\mathrm{h}))\right] \tag{27}$$

$$C_m = \frac{1}{-2\nu\beta_m\, U\, F(\beta_m)} \left[\frac{N(h_{s_2} - \beta_m)}{(h_{s_1} + \beta_m)(\cosh(2\beta_m \mathrm{h}) + \sinh(2\beta_m \mathrm{h}))} + L\right] \tag{28}$$

Special case and Numerical calculations

Setting

$$f(r) = \delta(r - r_0),\ r_0 = 1.5. \tag{29}$$

where, $\delta(r)$ is well known dirac delta function of argument r.

Applying finite Hankel transform as defined in eq.(13) to the eq.(29), one obtains

$$F(\beta_m) = \frac{r_0}{\beta_m}\left[\frac{J_0(\beta_m r_0)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r_0)}{Y_0'(\beta_m b)}\right]$$

$$q(r, z) = \delta(r - r_0)\delta(z - z_0),\ z_0 = 0 \tag{30}$$

$$\bar{q}(\beta_m, z) = \frac{r_0}{\beta_m}\delta(z - z_0)\left[\frac{J_0(\beta_m r_0)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r_0)}{Y_0'(\beta_m b)}\right]$$

$a = 1m, b = 2m,\ h_{s_1} = 13 \text{ and } h_{s_2} = 17.$

For thick plate $h = 0.25$ and for thin plate $h = 0.1$.

**Material Properties**

The numerical calculation has been carried out for a copper (pure) circular plate with the material properties defined as,

Thermal diffusivity $\alpha = 112.34\times 10^{-6}\, m^2 s^{-1}$,
Specific heat $c_\rho = 383 J/Kg$,
Thermal conductivity $k = 386$ W/m K,

Shear modulus $G = 48\ G\,pa$,
Poisson ratio $\vartheta = 0.3$.

**Roots of Transcendental Equation**

The $\beta_1 = 3.1965,\ \beta_2 = 6.3123,\ \beta_3 = 9.445,\ \beta_4 = 12.5812,\ \beta_5 = 15.7199$ are the roots of transcendental equation $\frac{J_1(\beta_m a)}{J_1(\beta_m b)} - \frac{Y_1(\beta_m a)}{Y_1(\beta_m b)} = 0$ . The numerical calculation and the graph has been carried out with the help of mathematical software Matlab.

**Discussion**

In this paper a thick $(M \neq 0)$ and thin $(M = 0)$ annular disc is considered which is free from traction and determined the expressions for temperature, displacement and stresses due to internal heat generation within it and we compute the effects of Michell function on the thickness of annular disc in terms of stresses along radial direction. As a special case mathematical model is constructed for considering copper (pure) annular disc with the material properties specified above.

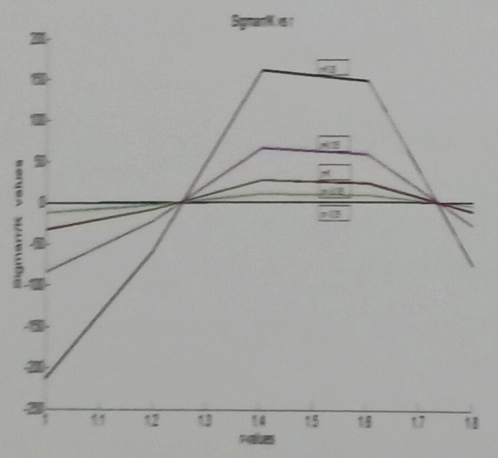

Figure 1 Radial stress $\frac{\sigma_{rr}}{K}$ for thick annular disc (M≠ 0).

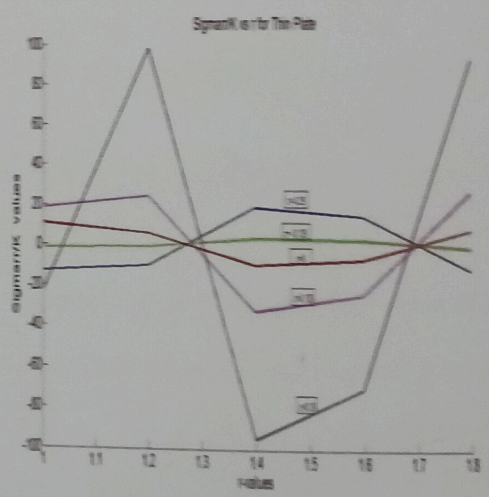

Figure 2 Radial stress $\frac{\sigma_{rr}}{K}$ for thin annular disc (M=0).

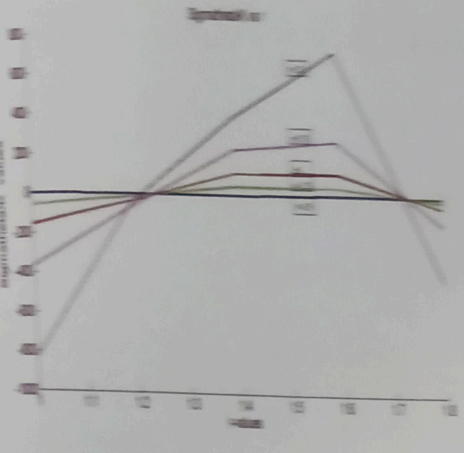

Figure3 Angular stress $\frac{\sigma_{\theta\theta}}{K}$ for thick annular disc ($M \neq 0$).

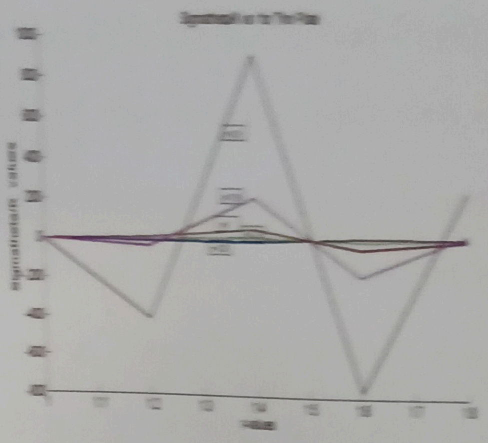

Figure4 Angular stress $\frac{\sigma_{\theta\theta}}{K}$ for thin annular disc ($M=0$).

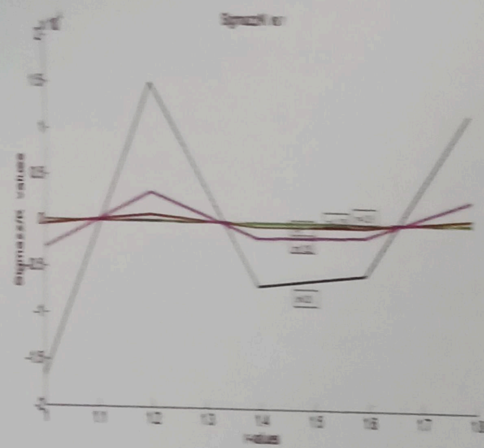

Figure 5 Axial stress $\frac{\sigma_{zz}}{K}$ for thick annular disc ($M \neq 0$)

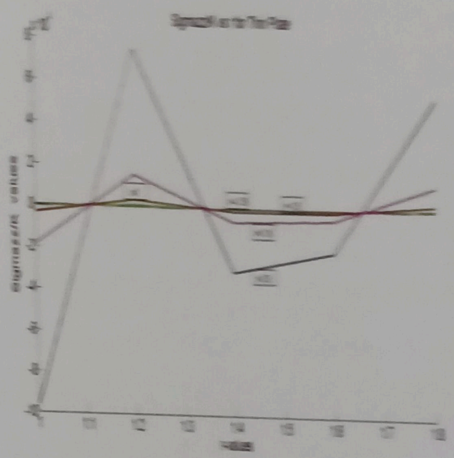

Figure 6 Axial stress $\frac{\sigma_{zz}}{K}$ for thin annular disc ($M=0$)

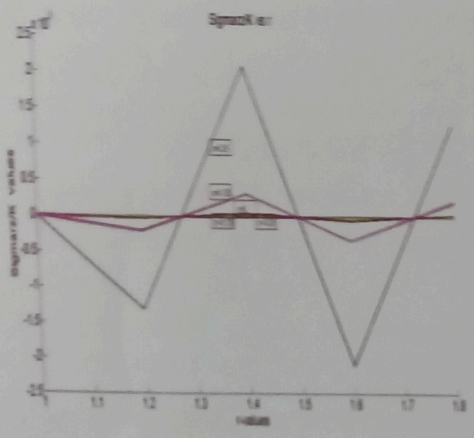

Figure 7 Stress $\frac{\sigma_{rz}}{K}$ for thick annular disc ($M \neq 0$)

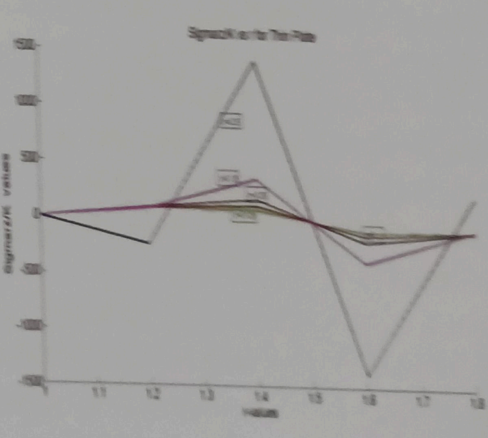

Figure 8 Stress $\frac{\sigma_{rz}}{K}$ for thin annular disc ($M=0$)

From figure 1 and 2, it is observed that due to Michell function the radial stress function $\frac{\sigma_{rr}}{K}$ increases along radial direction with the thickness of a thick annular disc.

From figure 3 and 4, it is observed that due to Michell function the angular stress function $\frac{\sigma_{\theta\theta}}{K}$ vary with the thickness of a thick annular disc along radial direction.

From figure 5 and 6, it is observed that due to Michell function the axial stress function $\frac{\sigma_{zz}}{K}$ is inversely vary the thickness of a thick annular disc along radial direction.

From figure 7 and 8, it is observed that due to Michell function the stress function $\frac{\sigma_{rz}}{K}$ vary with the thickness of a thick annular disc along radial direction.

It means we find out that due to Michell function the radial stress function $\frac{\sigma_{rr}}{K}$, angular stress function $\frac{\sigma_{\theta\theta}}{K}$ and the stress function $\frac{\sigma_{rz}}{K}$ vary with the thickness of a thick annular disc along radial direction whereas the axial stress function $\frac{\sigma_{zz}}{K}$ is inversely vary the thickness of a thick annular disc along radial direction.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thick annular disc and base of furnace of boiler of a thermal power plant and gas power plant and the measurement of aerodynamic heating.

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